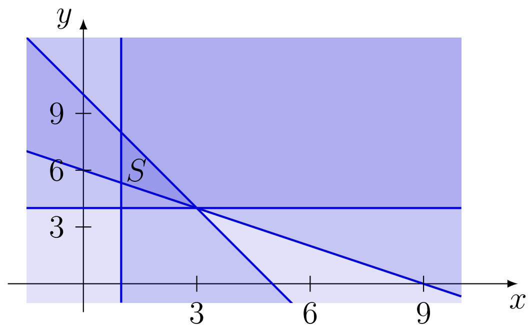
Math 102 Exam #5 Key (Spring 2012)

1a. The feasible region S is graphed below. Vertices are at $(3,4)$, $(1,5\frac{1}{3})$, and $(1,8)$.



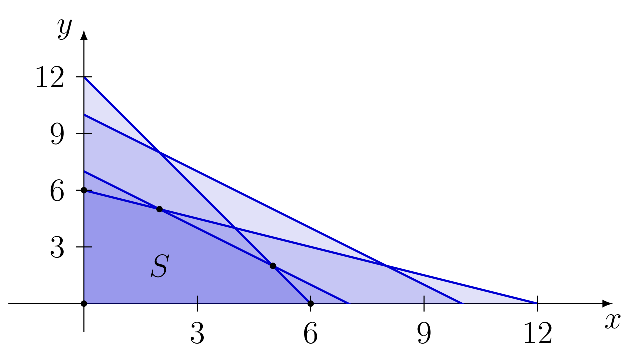
1b. $P(3,4) = 2.20(3) + 1.65(4) = 13.2$, $P(1,5\frac{1}{3}) = 11$, $P(1,8) = 15.4$. Maximum value is 15.4, minimum value is 11.

2a. Let x be the number of acres of wheat planted, and y the number of acres of rye. The constraints are

$$\begin{cases} x + y \leq 10 \\ x + y \geq 7 \\ 200x + 100y \leq 1200 \\ x + 2y \leq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

2b. $P = 500x + 300y$

2c. The feasible region S is where all shadings overlap. Vertices: $(0,0)$, $(0,6)$, $(6,0)$, $(2,5)$, $(5,2)$. Vertex $(2,5)$ is at the intersection of $x + y = 7$ and $x + 2y = 12$, and vertex $(5,2)$ is where $x + y = 7$ and $2x + y = 12$ intersect.



2d. Evaluate P at each vertex:

$$\begin{aligned} P(0,0) &= 500(0) + 300(0) = 0 \\ P(0,6) &= 500(0) + 300(6) = 1800 \\ P(6,0) &= 500(6) + 300(0) = 3000 \\ P(2,5) &= 500(2) + 300(5) = 2500 \\ P(5,2) &= 500(5) + 300(2) = 3100 \end{aligned}$$

From this it can be seen that 5 acres of wheat and 2 acres of rye should be planted for a maximum profit of \$3100.

3. Let x and y denote the measures of the two angles. Then $x + y = 180°$ and $x - y = 88°$. Adding the two equations gives $2x = 268°$, or $x = 134°$, and then $y = 180° - x = 46°$. That is, the angles are $134°$ and $46°$.

4a. $\overleftrightarrow{BH}$ or $\overleftrightarrow{DE}$, among other representations.

4b. $\overline{EF}$

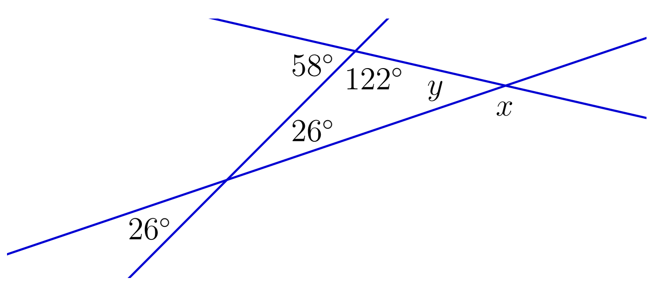
4c. $\{F\}$

4d. $\overrightarrow{BE}$

4e. $\varnothing$ or $\{ \; \}$

4f. $\overset{\circ\!\!-\!\!\circ}{DE}$

5. The interior angles of a triangle add up to $180°$, and x is supplementary to one of these angles—labeled y in the figure below. We have $y = 180° - 122° - 26° = 32°$, and therefore $x = 180° - y = 148°$.



6. $\dfrac{x}{8} = \dfrac{7}{12} \;\Rightarrow\; x = \dfrac{56}{12} = \dfrac{14}{3}$ and $\dfrac{y}{3.2} = \dfrac{12}{7} \;\Rightarrow\; y = 3.2\left(\dfrac{12}{7}\right) = \dfrac{192}{35}.$

7. Area of the big circle is $A_1 = \pi(16)^2 = 256\pi$. Area of one of the smaller circles is $A_2 = \pi(8)^2 = 64\pi$. Thus the area of the shaded region is

$$A = A_1 - 2A_2 = 256\pi - 2(64\pi) = 128\pi \approx 402.12 \text{ cm}^2.$$