

MATH 102 EXAM #3 KEY (SPRING 2012)

1a. $6 + 7 = 1$

1b. $4 - 10 = 6$

1c. $4 + 4 = 2$

2a. $39 \div 8$ has remainder 7, so $39 \equiv 7(\text{mod } 8)$.

2b. $4 + 7 = 11$, and $11 \div 6$ has remainder 5, so $4 + 7 \equiv 5(\text{mod } 6)$.

2c. $-18 + 4(5) = 2$, and since $2 \equiv 2(\text{mod } 5)$, it follows that $-18 \equiv 2(\text{mod } 5)$.

3a. $6 + 5 = 11$, and since $11 \div 8$ has remainder 3, it follows that $6 + 5 \equiv 3(\text{mod } 8)$. Solution set is $\{6\}$.

3b. For $x = 0, 1, 2, 3, 4, 5$ we have $3x = 0, 3, 6, 9, 12, 15$. Now, dividing 0, 6, 12 by 6 gives a remainder of 0, while dividing 3, 9, 15 by 6 gives a remainder of 3. Since a remainder of 2 never results, there is no solution to $3x \equiv 2(\text{mod } 6)$. Solution set is \emptyset .

3c. In the table below it can be seen that $2x \div 10$ will have a remainder of 4 when $x = 2, 7$. Solution set is thus $\{2, 7\}$.

x	0	1	2	3	4	5	6	7	8	9
$2x$	0	2	4	6	8	10	12	14	16	18
$2x \div 10$ rem.	0	2	4	6	8	0	2	4	6	8

4a. The cycle is 18 days long. $185 \div 18$ has remainder 5, so advance 5 days forward through the cycle starting from Day 1 of the Rivendell \rightarrow Isengard journey to find that the peddler is in Day 2 of his rest at Isengard.

4b. The cycle is 18 days long. $500 \div 18$ has remainder 14, so advance 14 days backward through the cycle starting from Day 1 of the Rivendell \rightarrow Isengard journey to find that the peddler is in Day 1 of his rest at Isengard.

5a. $3234 = 2 \cdot 3 \cdot 7^2 \cdot 11$

5b. $4959 = 3^2 \cdot 19 \cdot 29$

6a. $\text{GCD} = 21$ and $\text{LCM} = 58,212$

6b. $\text{GCD} = 2 \cdot 3 = 6$ and $\text{LCM} = 2^3 \cdot 3^3 \cdot 5^2 \cdot 7 = 37,800$

7. We find the least common multiple of the three numbers to arrive at 2,590,770 days.

8a. Let $N = 4.\bar{5}$. Then $10N = 45.\bar{5}$, and $9N = 10N - N = 45.\bar{5} - 4.\bar{5} = 41$ shows that $N = 41/9$

8b. Let $N = 2.0\bar{8}2$, so $10N = 20.\bar{8}2$ and $1000N = 2082.\bar{8}2$. Now, $990N = 1000N - 10N = 2062$, and therefore $N = 2062/990 = 1031/495$.

9a. Irrational

9b. Rational. Number can be written as $3.0\overline{12112111211212}$.

9c. Rational, because it terminates, you see.

10a. Commutative property of addition

10b. Associative property of multiplication

11. $32\left(\frac{1}{16}x - \frac{1}{32}\right) = 32 \cdot \frac{1}{16}x - 32 \cdot \frac{1}{32} = 2x - 1.$