Math 102 Exam #2 Key (Spring 2012)

1.
- Closure: Fails, since $1 - 2 = -1$ and -1 is not a whole number.
- Associative: Fails, since $5 - (3 - 1) = 5 - 2 = 3$ and $(5 - 3) - 1 = 2 - 1 = 1$.
- Identity: Fails, since the only viable candidate is 0, but $0 - 2 \neq 2$.
- Inverse: Fails, since there is no identity element.
- Commutativity: Fails, since $1 - 2 \neq 2 - 1$.

2.
- Closure. Let a and b be whole numbers. Then a and b are integers, so $a - b$ is an integer by A1, and thus $|a - b|$ is an integer. But the absolute value of any real number is nonnegative, so $|a - b|$ must be an integer that's greater than or equal to 0; that is, $|a - b|$ must be a whole number. Closure holds.
- Associativity. $1 \ominus (2 \ominus 3) = 1 \ominus 1 = 0$, but $(1 \ominus 2) \ominus 3 = 1 \ominus 3 = 2$. Associativity fails.
- Identity. For any whole number a we have $a \ominus 0 = |a - 0| = |a| = a$ and $0 \ominus a = |0 - a| = | - a| = a$ using A6, since $a - 0 = a + 0$ and $0 - a = 0 + (-a)$. So the identity element is 0. Identity property holds!
- Inverse. For any whole number a we have $a \ominus a = |a - a| = |0| = 0$, so a is its own inverse. Inverse property holds.
- Commutativity. For any whole numbers a and b, $a \ominus b = |a - b| = |b - a| = b \ominus a$. Commutativity holds.

3.
- Closure: Holds, since table contains no objects that aren't elements of the system.
- Associativity: Fails, since $\vee \otimes (\vee \otimes \cap) = \vee \otimes \vee = \sqcup$ whilst $(\vee \otimes \vee) \otimes \cap = \sqcup \otimes \cap = \cap$.
- Identity: Holds. Identity element is $\sqcup$.
- Inverse: Holds, since each element is its own inverse.
- Commutativity: Fails since $\cap \otimes \vee \neq \vee \otimes \cap$.

4.
- Closure: Holds, since table contains no objects that aren't elements of the system.
- Associativity: Fails since $\bowtie \boxdot (\perp \boxdot \curlyvee) = \bowtie \boxdot \bowtie = \perp$ whilst $(\bowtie \boxdot \perp) \boxdot \curlyvee = > \boxdot \curlyvee = \curlyvee$.
- Identity: Holds. Identity element is $>$.
- Inverse: Fails, since $\ltimes$ and $\curlyvee$ have no inverse (although $>$ is its own inverse, and $\bowtie$ and $\perp$ are inverses).
- Commutativity: Holds, since there is symmetry about the table's diagonal.

5a. $(4 \times 7^1) + (6 \times 7^0) + (3 \times 7^{-1}) = 34.\overline{428571}$

5b. $(12 \times 12^0) + (1 \times 12^{-1}) + (13 \times 12^{-2}) = 12.1736\overline{1}$

6a. 43.4_5

6b. 11.61_{12}

7a. 32.100_4

7b. 37.375_8

8a. $403_7 \div 6_7 = 45.111_7... = 45.\bar{1}_7$

$$
\begin{array}{r}
45.111_7 \\
6_7\overline{)\,403.000_7} \\
33 \\
\hline
43 \\
42 \\
\hline
10 \\
6 \\
\hline
10 \\
6 \\
\hline
10
\end{array}
$$

8b. $4233_8 \div 23_8 \approx 163.745_8$

$$
\begin{array}{r}
163.74503_8 \\
23_8\overline{)\,4233.00000_8} \\
23 \\
\hline
173 \\
162 \\
\hline
113 \\
71 \\
\hline
220 \\
205 \\
\hline
130 \\
114 \\
\hline
140 \\
137 \\
\hline
100 \\
71 \\
\hline
7
\end{array}
$$

9. $1010\ 1000\ 0101\ 1101_2$