# Math 102 Exam #2 Key (Fall 2010)

**1.** Closure: Fails, since $1-2=-1$ and $-1$ is not a positive integer. Associative: Fails, since $5-(3-1)=5-2=3$ and $(5-3)-1=2-1=1$. Identity: Fails, since the only viable candidate is 0, but $0-2\neq 2$. Inverse: Fails, since there is no identity element. Commutativity: Fails, since $1-2\neq 2-1$.

**2.** Closure: Holds. Let $a$ and $b$ be even integers. Then $a/2$ and $b/2$ are integers. It follows that $(a+b)/2$ is an integer since $(a+b)/2=a/2+b/2$, and $a/2+b/2$ must be an integer by A1. Since $(a+b)/2$ is an integer, $a+b$ is an *even* integer. Associativity: Holds by A4, since integers are reals. Identity: Holds. By A6 the even integer 0 is the identity element. Inverse: Holds. Let $a$ be an even integer. Then $a/2$ is an integer. Now, $-a/2=-1\cdot a/2$, which is a product of integers and must therefore be an integer by A1. Since $-a/2$ is an integer, it follows that $-a$ is an *even* integer. Finally, $a+(-a)=-a+a=0$ shows that $-a$, an even integer, is the *inverse* for $a$. Commutativity: Holds by A7, since integers are reals.

**3.** Closure: Holds, since table contains no objects that aren't elements of the system. Associativity: Fails, since $\vee\otimes(\vee\otimes\cap)=\vee\otimes\vee=\sqcup$ whilst $(\vee\otimes\vee)\otimes\cap=\sqcup\otimes\cap=\cap$. Identity: Holds, since $\sqcup$ is the identity element. Inverse: Holds, since each element is its own inverse. Commutativity: Fails since $\cap\otimes\vee\neq\vee\otimes\cap$.

**4.** Closure: Holds. Associativity: Fails since $\bowtie\boxdot(\perp\boxdot\curlyvee)=\bowtie\boxdot\bowtie=\perp$ whilst $(\bowtie\boxdot\perp)\boxdot\curlyvee=>\boxdot\curlyvee=\curlyvee$. Identity: Holds, $>$. Inverse: Fails, since $\ltimes$ and $\curlyvee$ have no inverse (although $>$ is its own inverse, and $\bowtie$ and $\perp$ are inverses). Commutativity: Holds, since there is symmetry about the table's diagonal.

**5a.** 6 **5b.** 9 **5c.** 1 **5d.** 1

**6a.** $41\div 9$ has remainder 5, so answer is 5.

**6b.** $4+7=11$, and since $11\div 6$ has remainder 5, the answer is 5.

**6c.** $8\cdot 7=56$, and since $56\div 5$ has remainder 1, the answer is 1.

**6d.** $5-12=-7$, and since $-7\div 5$ has remainder 3, the answer is 3.

**7a.**

| $x$ | $x+5$ | $\frac{x+5}{8}$ rem. |
|---|---|---|
| 0 | 5 | 5 |
| 1 | 6 | 6 |
| 2 | 7 | 7 |
| 3 | 8 | 0 |
| 4 | 9 | 1 |
| 5 | 10 | 2 |
| $\boxed{6}$ | 11 | 3 |
| 7 | 12 | 4 |

Solution is 6.

**7b.**

| $x$ | $2x$ | $\frac{2x}{6}$ rem. |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 2 | 2 |
| 2 | 4 | 4 |
| 3 | 6 | 0 |
| 4 | 8 | 2 |
| 5 | 10 | 4 |

There is no solution.

**7b.**

| $x$ | $4x$ | $\frac{4x}{10}$ rem. |
|---|---|---|
| 0 | 0 | 0 |
| $\boxed{1}$ | 4 | 4 |
| 2 | 8 | 8 |
| 3 | 12 | 2 |
| 4 | 16 | 6 |
| 5 | 20 | 0 |
| $\boxed{6}$ | 24 | 4 |
| 7 | 28 | 8 |
| 8 | 32 | 2 |
| 9 | 36 | 6 |

Solutions are 1 and 6.

**8a.** This is an 18-day cycle, and since $175\div 18$ has a remainder of 13, we find that 175 days from now the peddler will be in the 3rd day of rest at Bree.

**8b.** $400\div 18$ has remainder 4, so be go *backward* 4 days in the cycle to find the peddler in the 1st day of travel from Bree to Rivendell.