

- A1) For any $a, b \in \mathbb{Z}$, $a + b \in \mathbb{Z}$ and $a \cdot b \in \mathbb{Z}$
- A2) For any $a, b \in \mathbb{Q}$, $a + b \in \mathbb{Q}$ and $a \cdot b \in \mathbb{Q}$
- A3) For any $a, b \in \mathbb{R}$, $a + b \in \mathbb{R}$ and $a \cdot b \in \mathbb{R}$
- A4) For any $a, b, c \in \mathbb{R}$, $a + (b + c) = (a + b) + c$
- A5) For any $a, b, c \in \mathbb{R}$, $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
- A6) For any $a \in \mathbb{R}$, $a + 0 = a = 0 + a$ and $a \cdot 1 = a = 1 \cdot a$
- A7) For any $a, b \in \mathbb{R}$, $a + b = b + a$
- A8) For any $a, b \in \mathbb{R}$, $a \cdot b = b \cdot a$

1. 25 pts. Determine which of the five properties of a commutative group hold for the mathematical system consisting of the set of positive integers under the operation of subtraction. Use the axioms given above where appropriate.
2. 25 pts. Determine which of the five properties of a commutative group hold for the mathematical system consisting of the set of even integers under the operation of addition. Use the axioms given above where appropriate.
3. 25 pts. For the given mathematical system determine which of the five properties of a commutative group hold. If a property holds, explain why. If a property fails, give a counterexample.

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\vee	\vee	\sqcup	\cap
\cap	\cap	\vee	\sqcup

4. 25 pts. For the given mathematical system determine which of the five properties of a commutative group hold. If a property holds, explain why. If a property fails, give a counterexample.

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\times	\boxtimes	\perp	\times	γ	γ
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5. 5 pts. each Determine the sum or difference in the indicated clock arithmetic.
 - (a) $11 + 7$ in clock-12 arithmetic
 - (b) $8 - 11$ in clock-12 arithmetic
 - (c) $3 + 4$ in clock-6 arithmetic
 - (d) $2 - (3 + 5)$ in clock-7 arithmetic
6. 5 pts. each Determine what each is congruent to in the indicated modulo system.
 - (a) $41, \text{ mod } 9$
 - (b) $4 + 7, \text{ mod } 6$
 - (c) $8 \cdot 7, \text{ mod } 5$
 - (d) $5 - 12, \text{ mod } 5$
7. 10 pts. each Determine all nonnegative solutions that are less than the modulus.
 - (a) $x + 5 \equiv 3 \pmod{8}$
 - (b) $2x \equiv 1 \pmod{6}$
 - (c) $4x \equiv 4 \pmod{10}$
8. 10 pts. each A Middle-earth pipeweed peddler drives a wagon according to the following schedule: ride 3 days from Bree to Rivendell; rest 1 day at Rivendell; ride 4 days from Rivendell to Isengard; rest 2 days at Isengard; ride 5 days back to Bree; rest 3 days at Bree. Then the cycle begins anew. If the peddler is starting his trip to Isengard today, what will he be doing...
 - (a) 175 days from today?
 - (b) 400 days ago?