Math 102: Chapter 5 Supplementary Exercises

Chapter 5 is all about real numbers (as opposed to those, you know, "fake" numbers). Loosely speaking, in terms of our base-10 (decimal) numeration system, a **real number** is a sequence of numbers from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, along with a decimal point to establish the place values that the numbers occupy. Of course, there are an infinite number of ways that the numbers from 0 to 9 can be arranged in a finite or infinite sequence, and the decimal point can go anywhere between two numbers. The real numbers can be formally "constructed" using the set of rational numbers $\mathbb{Q}$ (which are in turn constructed from the set of integers $\mathbb{Z}$), but the process is rather complicated.

In section 5.1 the prime numbers are first considered, which are the "building blocks" of the natural numbers. All natural numbers are integers, which are discussed in section 5.2; and all integers are rational numbers, which are discussed in section 5.3. The real numbers that are not rational are called irrational, and they're the topic of section 5.4. Finally, all the rational and irrational numbers put together constitute the set of real numbers, the topic of section 5.5. Containing the set of real numbers $\mathbb{R}$ is the even bigger set of "complex numbers" $\mathbb{C}$, which will not be examined in this course.

1. Find the prime factorization of the number
   (a) 30,429
   (b) 99,789
   (c) 287,375
   (d) 347,072
   (e) 11,111
   (f) 11,113

2. Find the GCD and LCM of the numbers. On the exam it will be required that you show your work.
   (a) 4116 and 2646
   (b) 9295 and 760
   (c) 8888 and 6666
   (d) 72, 160, and 212
   (e) 2352, 2268, and 7560
   (f) 2520, 5292, and 21,609

3. Planet Ziltoid goes once around its sun every 507 days, while planet Arizonastan completes an orbit in 390 days and distant planet Everbitter's year is 1022 days long. If all three planets are in alignment today, in how many days will they all be in alignment again in the same place?

4. Express each repeating decimal number as a ratio (i.e. fraction) of integers. On an exam it will be required that you show your work.
   (a) 6.11111...
   (b) 5.284848484
   (c) 0.915915915915
   (d) 0.771313131313
   (e) 10.0042424242
   (f) 4.99999...

5. Determine whether the number is rational or irrational. Identify any repeating element.
   (a) 18.919919991999919999919999991...
   (b) 3.0121121112112121211211121212...
   (c) 0.265335533355533335553333355555
   (d) $\sqrt{4.00040001}$
   (e) $\sqrt{3} \cdot \sqrt{8}$ (combine radicals first)
   (f) $\sqrt{2} \cdot \sqrt{18}$ (combine radicals first)

6. Change one expression to the other, stating the property of real numbers being used at each step.
   (a) $2x + (15 + 7x)$ to $9x + 15$
   (b) $10y + 4(3 + 5y)$ to $30y + 12$