Math 102: Chapter 4 Supplementary Exercises Answers

1. Convert each numeral to Hindu-Arabic.

   (a) 7,213,540

   (b) 1,959

   (c) 1,629,448

   (d) 3,999,794

   (e) 311,852

   (f) 683,735

   (g) 215

   (h) 9,070

   (i) 1,406

   (j) $11(20 \times 18) + 4(20) + 16 = 4,056$

   (k) $10(20 \times 18) + 5(20) + 7 = 3,707$

   (l) $3(20^3 \times 18) + 2(20 \times 18) + 15(20) = 433,020$

   (m) $28(60) + 1 = 1,681$

   (n) $11(60^2) + 7(60) + 45 = 40,065$

   (o) $11(60^3) + 10(60^2) + 10(60) + 2 = 2,412,602$

2. Convert each Hindu-Arabic numeral to the indicated numeration system.

   (a) E = 𓆼𓆼𓎆
   R = MMX
   C = 二
千
零
一
十
   M = 𝋥
𝋪
𝋪
   G = ͵βι
   B = 𒌋𒌋𒌋𒁹𒁹𒁹 𒌋𒌋𒌋

   (b) E = 𓆼𓆼𓆼𓆼𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺
   R = $\overline{\text{IV}}$DCCCXLIX
   C = 四
千
八
百
四
十
九
   M = 𝋭
𝋨
𝋩
   G = ͵δωμθ
   B = 𒁹 𒌋𒌋 𒌋𒌋𒌋𒌋𒌋𒇲𒁹

   (c) E = 𓂭
   R = $\overline{\text{X}}$
   G = ͵ι
   B = 𒁹𒁹 𒌋𒌋𒌋𒌋𒁹𒁹𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋
   M = 𝋡
𝋧
𝋮
𝋠

   (d) E = 𓂭𓂭𓂭𓂭𓂭𓂭𓆼𓆼𓆼𓆼𓆼𓆼𓆼𓆼𓆼𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺
   R = $\overline{\text{LXIX}}$CMXCV
   G = ͵ξ͵θϡϟε
   B = 𒁹𒁹 𒌋𒌋𒌋𒌋𒁹𒁹𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋
   M = 𝋡
𝋧
𝋮
𝋠

   (e) E = 𓆐
   R = $\overline{\text{C}}$
   G = ͵ρ
   B = 𒌋𒌋𒌋𒇲𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒁹𒁹𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋
   M = 𝋭
𝋱
𝋮
𝋠

(f) E = 𓆐𓆐𓆐𓆐𓆐𓆐𓆐𓆼𓆼𓍢𓍢𓍢𓍢𓍢𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺

R = $\overline{\text{DCCII}}$DLV

G = ${}^{\iota}\psi{}^{\iota}\beta\phi\nu\epsilon$

B = 𒌋𒌋𒌋 𒌋𒁹𒁹𒁹𒁹𒁹 𒌋𒇲𒁹 𒌋𒁹𒁹𒁹𒁹𒁹

M = 𝋤
𝋱
𝋫
𝋩
𝋯

(g) E = 𓁨

R = $\overline{\text{M}}$

B = 𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒇲𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒁹𒁹𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋

M = 𝋤
𝋱
𝋫
𝋩
𝋯

(h) E = 𓁨𓁨𓁨𓆐𓆐𓆐𓆐𓆐𓆐𓆐𓆐𓂭𓂭𓂭𓂭𓂭𓍢𓍢𓍢𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺

R = $\overline{\text{MMMDCCCL}}$CCCLXXXIV

B = 𒌋𒌋𒇲𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒌋𒇲𒁹 𒌋𒌋𒌋𒁹𒁹𒁹 𒁹𒁹𒁹𒁹

M = 𝋡
𝋦
𝋮
𝋯
𝋩
𝋤

(i) E = 𓁨𓁨𓁨𓁨𓎆

B = 𒌋𒌋𒇲𒁹𒁹 𒌋𒌋𒌋𒁹 𒁹𒁹𒁹𒁹𒁹𒁹 𒌋𒌋𒌋𒌋𒌋

M = 𝋡
𝋧
𝋯
𝋫
𝋢
𝋪

3. Convert each to base-10.
   (a) $(5 \times 16^4) + (15 \times 16^3) + (12 \times 16^2) + (7 \times 16) + 10 = 392,314$
   (b) $(11 \times 12^4) + (10 \times 12^3) + (11 \times 12^2) + (10 \times 12) + 10 = 247,090$
   (c) $(2 \times 3^9) + (1 \times 3^8) + (0 \times 3^7) + (1 \times 3^6) + (2 \times 3^5) + (1 \times 3^4) + (0 \times 3^3) + (2 \times 3^2) + (0 \times 3) + 1 = 47,242$
   (d) $(5 \times 5^6) + (4 \times 5^5) + (1 \times 5^3) + (2 \times 5^2) + (4 \times 5) + 3 = 59,573$
   (e) $4^9 + 4^6 + 4^3 + 4^2 + 4 + 1 = 266,325$

4. Convert each to the indicated base.
   (a) $1,000,000 = (4 \times 12^5) + (0 \times 12^4) + (2 \times 12^3) + (8 \times 12^2) + (5 \times 12) + 4 = 402854_{12}$
   (b) $250,765 = (2 \times 7^6) + (0 \times 7^5) + (6 \times 7^4) + (3 \times 7^3) + (0 \times 7^2) + (4 \times 7) + 4 = 2063044_7$
   (c) $38,293 = (9 \times 16^3) + (5 \times 16^2) + (9 \times 16) + 5 = 9595_{16}$
   (d) $951 = (1 \times 3^6) + (0 \times 3^5) + (2 \times 3^4) + (2 \times 3^3) + (0 \times 3^2) + (2 \times 3) + 0 = 1022020_3$
   (e) $34,706,083 = (2 \times 16^6) + (1 \times 16^5) + (1 \times 16^4) + (9 \times 16^3) + (2 \times 16^2) + (10 \times 16) + 3 = 21192\text{A}3_{16}$
   (f) $306,888 = (1 \times 6^7) + (0 \times 6^6) + (3 \times 6^5) + (2 \times 6^4) + (4 \times 6^3) + (4 \times 6^2) + (4 \times 6) + 0 = 10324440_6$

5. Convert each to base-10.
   (a) $(7 \times 8) + 3 + (4 \times 8^{-1}) = 59.5$
   (b) $(10 \times 16) + 5 + (14 \times 16^{-1}) + (8 \times 16^{-2}) = 165.90625$
   (c) $2^6 + 2^3 + 2 + 1 + 2^{-1} + 2^{-3} = 75.625$
   (d) $(10 \times 12^{-3}) + (10 \times 12^{-4}) = 0.0062692901$
   (e) $2 + (4 \times 6^{-1}) + (5 \times 6^{-2}) = 2.80\bar{5}$

6. Convert each to the indicted base.
   (a) Fraction form: $2.5 = \dfrac{5}{2} = \dfrac{101_2}{10_2}$, where we simply convert 5 and 2 to base-2.
   Radix form: $2.5 = 2 + 1/2 \longrightarrow \underbrace{1}_{2^1=2}\ \underbrace{0}_{2^0=1}\ .\ \underbrace{1}_{2^{-1}=1/2} \longrightarrow 10.1_2$
   (b) Fraction form: $100.35 = \dfrac{10,035}{100} = \dfrac{2007}{20} = \dfrac{310012_5}{40_5}$
   Radix form: you can get the radix form the usual way, but you'll soon come to a point (starting at the 5^{-2} place) where a 3 seems to repeat. How can we be sure that the 3 repeats forever? Well, I wouldn't be so nasty as to give you a number with a digit that repeats more than three times but *doesn't* repeat forever, but for those who are curious here's a way around the problem: take the base-5 fraction form

above and get the radix form by performing long division: $400.133\ldots \longrightarrow 400.1\bar{3}_5$

$$
\begin{array}{r}
400.133\ldots \\
40\overline{)31012.000} \\
\underline{310} \\
01 \\
\underline{0} \\
12 \\
\underline{0} \\
120 \\
\underline{40} \\
300 \\
\underline{220} \\
300 \\
\underline{220} \\
300
\end{array}
$$

(c) $\frac{2}{3} = \frac{4}{6} \longrightarrow 0.4_6$ and $\frac{2}{3} = \frac{6}{9} = 0.6_9$

(d) $\frac{26}{49} = \frac{3}{7} + \frac{5}{49} = 0.35_7$. Just do the usual thing: divide $\frac{26}{49}$ by the largest place value that doesn't exceed the number, which is 7^{-1}, find that 7^{-1} "fits" into $\frac{26}{49}$ just 3 times, then subtract 3×7^{-1} away from $\frac{26}{49}$ to get a remainder of $\frac{5}{49}$. Next, divide the remainder by the next place value down, which is 7^{-2}, to see that 7^{-2} "fits" into the remainder exactly 5 times with *no* remainder. Done.

(e) $\frac{139}{144} = \frac{11}{12} + \frac{7}{144} \longrightarrow 0.\text{E}7_{12}$

(f) $19.125 \longrightarrow 10011.001_2 \longrightarrow 103.02_4 \longrightarrow 23.1_8 \longrightarrow 13.2_{16}$ (this is done fast by using the base-2 form to convert to bases 4, 8, and 16).

(g) No tricks here: 0.24_5

7. Convert directly to the base indicated.

(a) So, lump the base-2 digits into groups of four. Notice the leftmost group needs an extra couple of zeros so that it also has four digits. As shown in class, each group of *four* base-2 digits translates directly into *one* digit in base-16.

$$\underbrace{0011}_{3}\ \underbrace{0111}_{7}\ \underbrace{0101}_{5}\ \underbrace{0111}_{7}\ \underbrace{0000}_{0}\ \underbrace{1101}_{13}\ \underbrace{0101}_{5}\ \underbrace{0101}_{5}\ \underbrace{1110}_{14} \longrightarrow 37570\text{D}55\text{E}_{16}$$

(b) Let's have at it: $\underbrace{0111}_{7} . \underbrace{0011}_{3}\ \underbrace{0111}_{7} \longrightarrow 7.37_{16}$

(c) Each base-16 digit unpacks as four base-2 digits, with $\text{A} = 10 = 2^3 + 2 = 1010_2$ and so on...

$$\underbrace{\text{A} = 10}_{1010}\ \underbrace{\text{B} = 11}_{1011}\ \underbrace{\text{C} = 12}_{1100}\ \underbrace{\text{D} = 13}_{1101}\ \underbrace{\text{E} = 14}_{1110}\ \underbrace{\text{F} = 15}_{1111} \longrightarrow 1010\ 1011\ 1100\ 1101\ 1110\ 1111_2$$

(d) It's like magic: $\underbrace{7}_{0111}\ \underbrace{D}_{1101} . \underbrace{1}_{0001}\ \underbrace{F}_{1111} \longrightarrow 0111\ 1101 . 0001\ 1111_2$, or simply 1111101.00011111_2

8. There are more digits beyond the 2^{-9} place, but that's not your problem: $3.14 \approx 11.001000111_2$

9. $d_7 d_{41} d_{28} \longrightarrow (7 \times 60^2) + (41 \times 60) + 28 = 27{,}688$ and $152{,}373 = (42 \times 60^2) + (19 \times 60) + 33 \longrightarrow d_{42} d_{19} d_{33}$

10. Carry out the long division in the base indicated.

(a) $403_7 \div 6_7 = 45.111_7... = 45.\bar{1}_7$

$$\begin{array}{r}
45.111_7\\
6_7\,\overline{)\,403.000_7}\\
\underline{33}\\
43\\
\underline{42}\\
10\\
\underline{6}\\
10\\
\underline{6}\\
10
\end{array}$$

(b) $2404_5 \div 44_5 = 24.333_5... = 24.\bar{3}_5$

$$\begin{array}{r}
24.333_5\\
44_5\,\overline{)\,2404.000_5}\\
\underline{143}\\
424\\
\underline{341}\\
330\\
\underline{242}\\
330\\
\underline{242}\\
330
\end{array}$$

(c) $4233_8 \div 23_8 \approx 163.745_8$

$$\begin{array}{r}
163.74503_8\\
23_8\,\overline{)\,4233.00000_8}\\
\underline{23}\\
173\\
\underline{162}\\
113\\
\underline{71}\\
220\\
\underline{205}\\
130\\
\underline{114}\\
140\\
\underline{137}\\
100\\
\underline{71}\\
7
\end{array}$$

(d) $2340_5 \div 34_5 \approx 33.034_5$

$$\begin{array}{r}
33.03433_5\\
34_5\,\overline{)\,2340.00000_5}\\
\underline{212}\\
220\\
\underline{212}\\
300\\
\underline{212}\\
330\\
\underline{301}\\
240\\
\underline{212}\\
230\\
\underline{212}\\
130
\end{array}$$

(e) $503_6 \div 21_6 \approx 22.024_6$

$$\begin{array}{r}
22.02434_6\\
21_6\,\overline{)\,503.00000_6}\\
\underline{42}\\
43\\
\underline{42}\\
100\\
\underline{42}\\
140\\
\underline{124}\\
120\\
\underline{103}\\
130\\
\underline{124}\\
2
\end{array}$$