Math 101
Summer 2011
Exam #2

Name:

1. [15 pts.] Use a truth table to determine whether the statements $(p \to q) \wedge (q \to r)$ and $(p \to q) \to r$ are equivalent.

2. [10 pts.] Use DeMorgan's laws to write an equivalent statement for the sentence: "It is not the case that entropy always increases in an open thermodynamic system or radiocarbon dating is not reliable."

3. [10 pts.] Use the fact that $p \to q$ is equivalent to $\neg p \vee q$ to rewrite the statement "Either the clowns in Congress will listen to the people or the system doesn't work."

4. [10 pts.] Write the contrapositive of the statement "If you're not with us, then you're against us."

5. [15 pts.] Use truth tables or established logical equivalencies to determine which of the three statements are equivalent.

   i. The car needs a subwoofer, and either the car needs an amp or the car is new.
   ii. The car needs a subwoofer, and it is false that the car does not need an amp and the car is not new.
   iii. If the car needs a subwoofer, then the car needs an amp or the car is not new.

6. [10 pts.] Determine whether the argument is valid using a truth table:
   $p \to q$
   $\neg q$
   $\therefore \neg p$

7. [20 pts.] Translate the argument into symbolic form, then determine whether the argument is valid using a truth table: "The engineering courses are hard and the chemistry labs are long. If the chemistry labs are long, then the art tests are easy. Therefore the engineering courses are hard and the art tests are easy."

8. [10 pts. each] Use an Euler diagram to determine whether the syllogism is valid.

   (a) No wiener dogs are caterpillars.
   No caterpillars are four-legged.
   $\therefore$ No wiener dogs are four-legged.

   (b) All nutty professors wear tweed coats.
   Some tweed coat wearers are not stylish.
   $\therefore$ Some nutty professors are not stylish.

   (c) Some clowns are scary people.
   Some scary people are fascists.
   All fascists are clowns.
   $\therefore$ Some fascists are not scary people.

9. [10 pts.] Jeb finds an irregularly-shaped five-sided rock on his dirt farm. He labels each side and tosses the rock 100 times whilst tanked on moonshine. The results of his tosses are shown in the table below. Determine the empirical probability that the rock will display side 4 if tossed again.

Side	1	2	3	4	5
Frequency	32	18	15	13	22

10. [10 pts.] A traffic light is red for 25 sec, yellow for 5 sec, and green for 55 sec. What is the probability that when you reach the light it will be yellow?

11. [10 pts.] One card is selected at random from a deck of cards. Find the probability that the card selected is not a 5.

12. [10 pts.] A die is tossed. Find the odds against rolling a number less than 3.

13. [10 pts.] The odds in favor of Wendy winning a scholarship are 7:4. Find the probability that Wendy wins.

14. [10 pts.] Here's a grand little game: a 15-sided die is rolled once. If an even number comes up you win \$10; if a 1 or 3 comes up you lose \$6; if a 5, 7, 9, or 11 comes up you lose \$1; if a 13 comes up you lose \$50; and if a 15 comes up you break even. What's your expected value if you play this game?

15. [10 pts. each] A multiple-choice exam has five possible answers for each question. For each correct answer you get 10 points. For each wrong answer you lose 3 points. For answers left blank, no points are gained or lost.

   (a) If you don't know the correct answer to a question, is it to your advantage to guess? Explain.

   (b) If you don't know the correct answer to a question but can eliminate two possible choices, is it to your advantage to guess? Explain.