MATH 101
SPRING 2023 NAME:
EXAM 1

1. $\boxed{\text{10 pts. each}}$ Express each set in roster form. (Note: $\mathbb{N}$ is the set of natural or counting numbers.)
   (a) The set of integers between -5 and 5.
   (b) $A = \{x \mid x \in \mathbb{N} \text{ and } x < 9\}$
   (c) $B = \{x \mid 8x + 3 = 1\}$

2. $\boxed{\text{10 pts. each}}$ Express each set in set-builder notation.
   (a) $C = \{4, 5, 6, 7, 8, 9, 10, 11, 12\}$.
   (b) D is the set of natural numbers that are divisible by 3.

3. $\boxed{\text{5 pts. each}}$ State whether each statement is true or false. If false, give the reason.
   (a) $\{\#\} \in \{\$, \&, \%, @, \#, =\}$
   (b) $\{\zeta, \alpha\} \subset \{\alpha, \beta, \gamma, \delta, \epsilon, \zeta\}$
   (c) $\boxdot \subset \{\boxdot, \square, \boxdot, \boxslash, \boxdot, \boxast\}$

4. $\boxed{\text{10 pts.}}$ List all the proper subsets of the set $\{a, b, c\}$.

5. $\boxed{\text{10 pts. each}}$ Determine each set, given that

$$\begin{aligned} U &= \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \\ A &= \{1, 2, 4, 5, 8\} \\ B &= \{3, 4, 7\} \end{aligned}$$

   (a) $(A \cup B)'$
   (b) $A' \cup (A \cap B)$
   (c) $B' - A$

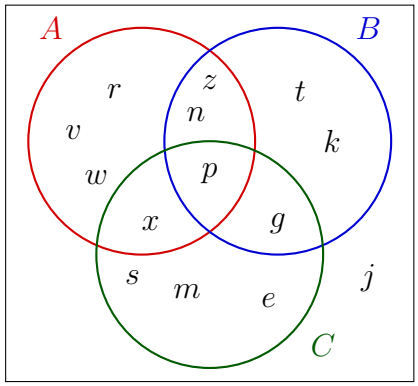
6. $\boxed{\text{10 pts. each}}$ Determine each set, given that

$$\begin{aligned} U &= \{x \mid x \in \mathbb{N} \text{ and } x < 10\} \\ A &= \{x \mid x \in \mathbb{N},\ x \text{ is odd, and } x < 10\} \\ B &= \{x \mid x \in \mathbb{N},\ x \text{ is even, and } x < 10\} \\ C &= \{x \mid x \in \mathbb{N} \text{ and } 6 < x < 10\} \end{aligned}$$

   (a) $(C' \cup A) \cap B$
   (b) $(A - B)' - C$

7. $\boxed{\text{15 pts.}}$ For $A = \{s, t\}$ and $B = \{4, 6, 8\}$, determine $A \times B$. Also, determine $n(A)$, $n(B)$, and $n(A \times B)$.

8. $\boxed{\text{10 pts.}}$ Use the Venn diagram below to determine the sets $A \cap (B \cup C)$ and $(A' \cup B) \cap C$.



9. $\boxed{\text{10 pts. each}}$ Using the textbook's method, determine whether the following expressions are equal for all sets A, B, and C.

   (a) $(A' \cap B)'$, $\quad A \cup B'$

   (b) $A \cup (B \cap C)'$, $\quad A' \cap (B \cup C)$

10. Three grain crops raised in the world are wheat, quinoa, and rice. A survey of 47 countries that raise grain yielded the following results:

    18 countries raised wheat
    16 countries raised quinoa
    12 countries raised rice
    9 raised wheat and quinoa
    3 raised quinoa and rice
    3 raised wheat and rice
    2 raised all three crops

    (a) $\boxed{\text{8 pts.}}$ Draw a Venn diagram illustrating the information given above.
    (b) $\boxed{\text{3 pts.}}$ How many countries raised none of the three crops?
    (c) $\boxed{\text{3 pts.}}$ How many countries raised exactly two of the crops?
    (d) $\boxed{\text{3 pts.}}$ How many raised wheat and quinoa, but not rice?
    (e) $\boxed{\text{3 pts.}}$ How many raised quinoa or rice, but not wheat?

11. $\boxed{\text{10 pts.}}$ Show the set $\{-5, -2, 1, 4, \ldots\}$ is infinite by placing it in a one-to-one correspondence with a proper subset of itself. Show the pairing of the general terms of the sets.