Math 101 Exam #3 Key (Summer 2012)

1. Probability of yellow light $= \frac{5}{85} = \frac{1}{17} \approx 0.0588.$

2. Probability of no 5 $= \frac{48}{52} = \frac{12}{13} \approx 0.9231$

3. $4:2$ against, or equivalently $2:1$ against.

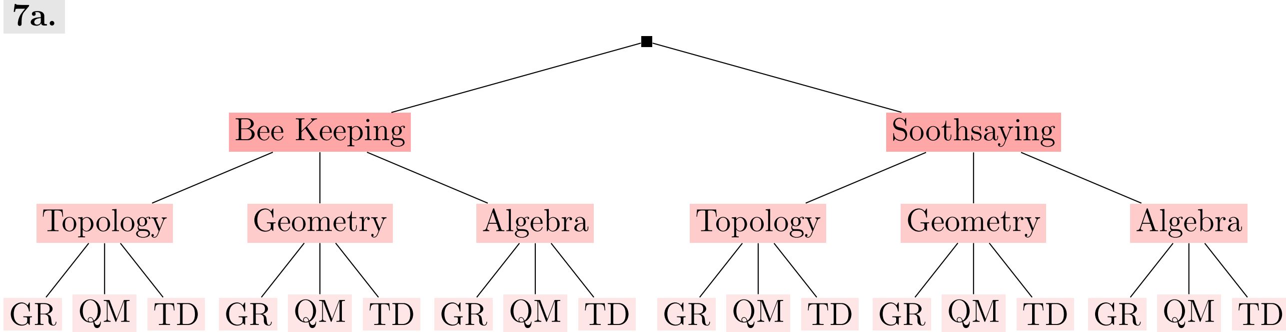
4. $\frac{7}{11}$

5. $\frac{7}{15}(\$10) + \frac{2}{15}(-\$6) + \frac{4}{15}(-\$1) + \frac{1}{15}(-\$50) + \frac{1}{15}(\$0) \approx \$0.27.$

6a. By guessing, expected value is $\frac{1}{5}(10) + \frac{4}{5}(-3) = -\frac{2}{5}$ points; so it would not pay off.

6b. By guessing, expected value is $\frac{1}{3}(10) + \frac{2}{3}(-3) = 1\frac{1}{3}$ points; so it now is worth guessing.

7a.



7b. $\frac{3}{18} = \frac{1}{6} \approx 0.167$

7c. $\frac{5}{18} \approx 0.278$

8. $\frac{16}{52} = \frac{4}{13} \approx 0.308$

9a. $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169} \approx 0.0059$

9b. $\frac{4}{52} \cdot \frac{4}{51} = \frac{1}{13} \cdot \frac{4}{51} = \frac{4}{663} \approx 0.0060$

10. $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{81}{4096} \approx 0.020$

11a. $P(\text{0-60} \mid \text{male}) = \frac{41}{171} \approx 0.240$

11b. $P(\text{female} \mid \text{over } 120) = \frac{14}{35} = \frac{2}{5} \approx 0.400$

11c. $P(\text{61-120 or over 120} \mid \text{male}) = P(\text{61-120} \mid \text{male}) + P(\text{over 120} \mid \text{male}) = \frac{109}{171} + \frac{21}{171} = \frac{130}{171} \approx 0.760$

12a. $26 \cdot 25 \cdot 24 \cdot 23 \cdot 10 \cdot 9 = 32,292,000$

12b. $26^4 \cdot 10^2 = 45,697,600$

13. $3^{10} = 59,049$

14. $\dfrac{12!}{(4!)(2!)} = 9,979,200$

15. ${}_{28}\mathrm{C}_{22} = 376,740$

16. ${}_{18}\mathrm{C}_6 \cdot {}_{12}\mathrm{C}_4 \cdot {}_8\mathrm{C}_2 = 18,564 \cdot 495 \cdot 28 = 257,297,040$

17. $\dfrac{{}_7\mathrm{C}_4}{{}_{13}\mathrm{C}_4} = \dfrac{35}{715} = \dfrac{7}{143} \approx 0.049$

18a. $\dfrac{{}_2\mathrm{C}_2}{{}_7\mathrm{C}_2} = \dfrac{1}{21} \approx 0.048$

18b. $P(\text{at least 1 car}) = P(\text{1 car or 2 cars}) = P(\text{1 car}) + P(\text{2 cars}) = \dfrac{{}_2\mathrm{C}_1 \cdot {}_5\mathrm{C}_1}{{}_7\mathrm{C}_2} + \dfrac{{}_2\mathrm{C}_2}{{}_7\mathrm{C}_2} = \frac{10}{21} + \frac{1}{21} = \frac{11}{21} \approx 0.524$

19. $\dfrac{{}_4\mathrm{C}_2 \cdot {}_4\mathrm{C}_2 \cdot {}_{44}\mathrm{C}_1}{{}_{52}\mathrm{C}_5} = \dfrac{(6)(6)(44)}{2,598,960} = \dfrac{1,584}{2,598,960} = \dfrac{33}{54,145} \approx 0.000609$

20. $P(2) = ({}_{12}\mathrm{C}_2)(0.004)^2(0.996)^{10} = 66(0.004)^2(0.996)^{10} \approx 0.00101$

21a. $P(5) = ({}_{10}\mathrm{C}_5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^5 = 252(0.2)^5(0.8)^5 \approx 0.0264$

21b. $P(9)+P(10) = ({}_{10}\mathrm{C}_9)\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)^1+({}_{10}\mathrm{C}_{10})\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^0 = 10\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)+\left(\frac{1}{5}\right)^{10} = 4.1984\times10^{-6}$

21c. $P(\geq 1) = 1 - P(0) = 1 - ({}_{10}\mathrm{C}_0)\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10} = 1 - \left(\frac{4}{5}\right)^{10} \approx 1 - 0.107 \approx 0.893$