Math 101 Exam #1 Key (Summer 2012)

1a. $\{3, 4, 5, 6\}$

1b. $\{2, 4, 6, 8, ...\}$

2a. $\{x \mid x \in \mathbb{N} \text{ and } 5 \leq x \leq 14\}$

2b. $\{x \mid x \in \mathbb{N} \text{ and } x \text{ is odd}\}$

3a. False, since $\{\#\}$ is actually a *subset* of $\{\$, \&, \%, @, \#, =\}$.

3b. True.

3c. False, since $\boxbox$ is actually an *element* of $\{\boxcircle, \square, \boxbox, \boxslash, \boxdot, \boxast\}$.

3d. False, since no set can be a proper subset of itself.

4. $\varnothing$, $\{\ogreaterthan\}$, $\{\sqcup\}$, $\{\times\}$, $\{\ogreaterthan, \sqcup\}$, $\{\ogreaterthan, \times\}$, $\{\sqcup, \times\}$

5a. $(A \cup B)' = \{7\}$

5b. $A' \cup (A \cap B) = \{3, 6, 7\} \cup \{2, 4\} = \{2, 3, 4, 6, 7\}$

5c. $\{1, 2, 4, 5, 8\} - \{1, 5, 7, 8\} = \{2, 4\}$

6. Here $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$, and $C = \{6, 7, 8, 9\}$, so

$$(C' \cup A) \cap B = \{1, 3, 5, 6, 7, 8, 9\} \cup \{2, 4, 6, 8\} = \{6, 8\}.$$

7. We have

$$A \times B = \{(4, q), (6, q), (8, q), (4, r), (6, r), (8, r)\},$$

and so $n(A) = 2$, $n(B) = 3$, and $n(B \times A) = 6$. (Notice that $n(A) \cdot n(B) = n(A \times B)$, which is not a coincidence.)

8. We have

$$A \cap (B \cup C) = \{r, v, w, x, p, n, z\} \cap \{z, n, p, x, s, m, e, g, t, k\} = \{z, n, p, x\},$$

and

$$\begin{aligned}(A' \cup B) \cap C &= (\{t, k, g, e, m, s, j\} \cup \{t, k, g, p, n, z\}) \cap \{g, e, m, s, x, p\} \\ &= \{t, k, g, e, m, s, p, n, z, j\} \cap \{g, e, m, s, x, p\} \\ &= \{g, e, m, s, p\}\end{aligned}$$

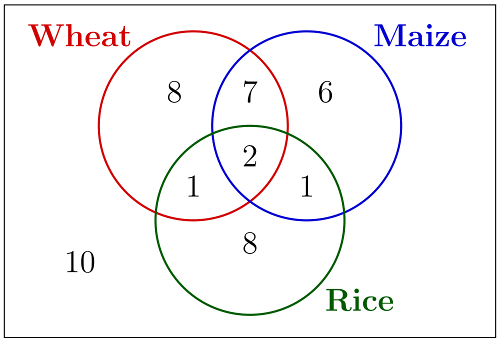
9a. Referring to the regions of a two-set Venn diagram, we find that

$$(A' \cap B)' = \{\text{I, II, IV}\} = A \cup B'.$$

9b. Referring to the regions of a three-set Venn diagram,

$$A \cup (B \cap C)' = \{\text{I, II, III, IV, V, VII, VIII}\} \neq \{\text{III, VI, VII}\} = A' \cap (B \cup C).$$

10a.



10b. $43 - 33 = 10$

10c. $8 + 8 + 6 = 22$

10d. 7

10e. $8 + 1 + 6 = 15$

11. For n any natural number we have

$$\begin{aligned} 3 &\longrightarrow 8 \\ 8 &\longrightarrow 13 \\ 13 &\longrightarrow 18 \\ 18 &\longrightarrow 23 \\ &\vdots \\ 5n - 2 &\longrightarrow 5n + 3 \end{aligned}$$

12. For n any natural number we have

$$\begin{aligned} 1 &\longrightarrow 1 \\ 3 &\longrightarrow 2 \\ 5 &\longrightarrow 3 \\ 7 &\longrightarrow 4 \\ &\vdots \\ 2n - 1 &\longrightarrow n \end{aligned}$$