Math 101 Exam #2 Key (Summer 2011)

1.

p	q	r	$(p \to q) \wedge (q \to r)$	$(p \to q) \to r$
1	1	1	1	1
1	1	0	0	0
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	0	0
0	0	1	1	1
0	0	0	1	0

Not equivalent

2. Let p be "Entropy always increases in an open thermodynamic system," and let q be "Radiocarbon dating is reliable." The given statement is then $\neg(p \vee \neg q)$, and by DeMorgan's Laws this becomes $\neg p \wedge q$, which translates as "Entropy does not always increase in an open thermodynamic system and radiocarbon dating is reliable." (This happens to be factually true, by the way, but it is not our concern.)

3. Let p be "The clowns in Congress will listen to the people," and q be "The system does work." Given statement is thus $p \vee \neg q$, which by the given equivalency becomes $\neg p \to \neg q$ and translates as "If the clowns in Congress don't listen to the people, then the system doesn't work."

4. "If you're not against us, then you're with us."

5. Let p be "The car needs a subwoofer," let q be "The car needs an amp," and let r be "The car is new." Then (i) is $p \wedge (q \vee r)$; and (ii) is $p \wedge \neg(\neg q \wedge \neg r)$; and (iii) is $p \to (q \vee \neg r)$. DeMorgan's Laws make clear that (i) $\Leftrightarrow$ (ii). We compare (i) and (iii) with a truth table to see that (i) $\not\Leftrightarrow$ (iii), from which it follows that (ii) $\not\Leftrightarrow$ (iii) also.

6.

p	q	$p \to q$	$\neg q$	$\neg p$
1	1	1	0	0
1	0	0	1	0
0	1	1	0	1
0	0	1	1	1

Valid

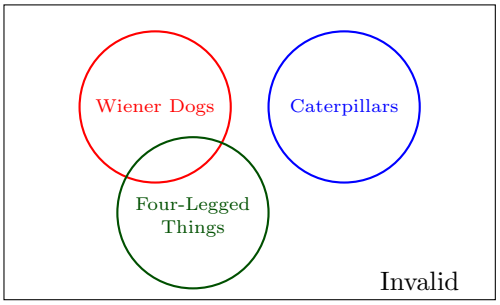
7. Let p be "The engineering courses are hard," let q be "The chemistry labs are long," and let r be "The art tests are easy." The argument is:

$$\begin{array}{l} p \wedge q \\ q \to r \\ \hline \therefore\ p \wedge r \end{array}$$

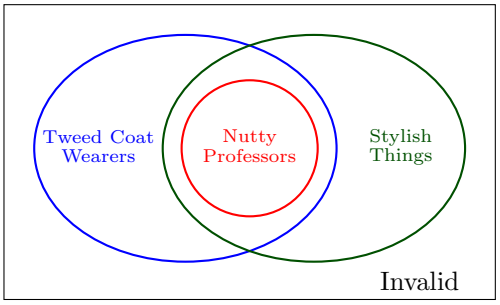
p	q	r	$p \wedge q$	$q \to r$	$p \wedge r$
1	1	1	1	1	1
1	1	0	1	0	0
1	0	1	0	1	1
1	0	0	0	1	0
0	1	1	0	1	0
0	1	0	0	0	0
0	0	1	0	1	0
0	0	0	0	1	0

Valid

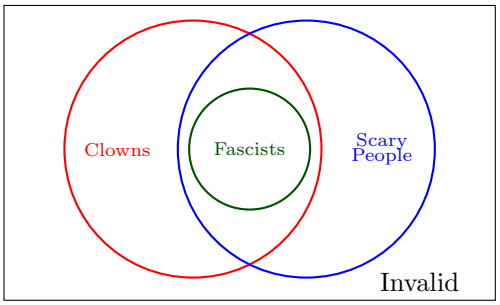
8a.



8b.



8c.



9. Probability of a 4 $= \dfrac{13}{100} = 0.13$

10. Probability of yellow light $= \dfrac{5}{85} \approx 0.0588.$

11. Probability of no 5 $= \dfrac{48}{52} \approx 0.9231$

12. $4 : 2$ against.

13. $\dfrac{7}{11}$

14. $\frac{7}{15}(\$10) + \frac{2}{15}(-\$6) + \frac{4}{15}(-\$1) + \frac{1}{15}(-\$50) + \frac{1}{15}(\$0) \approx \$0.27.$

15a. By guessing, expected value is $\frac{1}{5}(10) + \frac{4}{5}(-3) = -\frac{2}{5}$ points; so it would not pay off.

15b. By guessing, expected value is $\frac{1}{3}(10) + \frac{2}{3}(-3) = 1\frac{1}{3}$ points; so it now is worth guessing.