Math 101 Exam #1 Key (Summer 2011)

1a. $\{3, 4, 5, 6\}$

1b. $\{2, 4, 6, 8, ...\}$

2a. $\{x \mid x \in \mathbb{N} \text{ and } 5 \leq x \leq 14\}$

2b. $\{x \mid x \in \mathbb{N} \text{ and } x \text{ is odd}\}$

3a. False, since $\{\#\}$ is actually a *subset* of $\{\$, \&, \%, @, \#, =\}$.

3b. True.

3c. False, since $\boxbox$ is actually an *element* of $\{\boxcircle, \square, \boxbox, \boxslash, \boxdot, \boxast\}$.

3d. False, since no set can be a proper subset of itself.

4. $\varnothing$, $\{\ogreaterthan\}$, $\{\sqcup\}$, $\{\times\}$, $\{\ogreaterthan, \sqcup\}$, $\{\ogreaterthan, \times\}$, $\{\sqcup, \times\}$

5a. $(A \cup B)' = \{7\}$

5b. $A' \cup (A \cap B) = \{3, 6, 7\} \cup \{2, 4\} = \{2, 3, 4, 6, 7\}$

5c. $\{1, 2, 4, 5, 8\} - \{1, 5, 7, 8\} = \{2, 4\}$

6. Here $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$, and $C = \{6, 7, 8, 9\}$, so $(C' \cup A) \cap B = \{1, 3, 5, 6, 7, 8, 9\} \cup \{2, 4, 6, 8\} = \{6, 8\}$

7. $A \times B = \{(q, 4), (q, 6), (q, 8), (r, 4), (r, 6), (r, 8)\}$, $n(A) = 2$, $n(B) = 3$, $n(A \times B) = 6$. (Notice that $n(A) \cdot n(B) = n(A \times B)$, which is not a coincidence.)

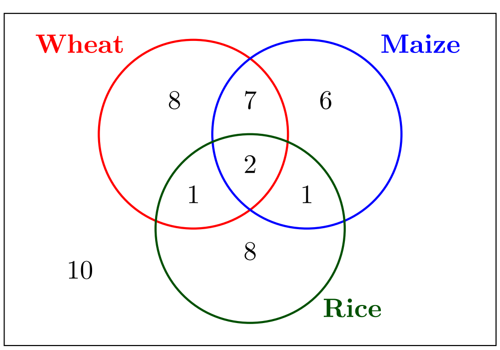
8. $A \cap (B \cup C) = \{r, v, w, x, p, n, z\} \cap \{z, n, p, x, s, m, e, g, t, k\} = \{z, n, p, x\}$, and

$$\begin{aligned}(A' \cup B) \cap C &= (\{t, k, g, e, m, s, j\} \cup \{t, k, g, p, n, z\}) \cap \{g, e, m, s, x, p\} \\ &= \{t, k, g, e, m, s, p, n, z, j\} \cap \{g, e, m, s, x, p\} \\ &= \{g, e, m, s, p\}\end{aligned}$$

9a. Referring to the regions of a two-set Venn diagram, we find that $(A' \cap B)' = \{\text{I, II, IV}\} = A \cup B'$.

9b. Referring to the regions of a three-set Venn diagram, $A \cup (B \cap C)' = \{\text{I, II, III, IV, V, VII, VIII}\} \neq \{\text{III, VI, VII}\} = A' \cap (B \cup C)$.

10a.



10b. $43 - 33 = 10$

10c. $8 + 8 + 6 = 22$

10d. 7

10e. $8 + 1 + 6 = 15$

11. For n any natural number we have

$$\begin{aligned} 3 &\longrightarrow 9 \\ 9 &\longrightarrow 15 \\ 15 &\longrightarrow 21 \\ 21 &\longrightarrow 27 \\ &\vdots \\ 6n-3 &\longrightarrow 6n+3 \end{aligned}$$

12. For n any natural number we have

$$\begin{aligned} 0 &\longrightarrow 1 \\ 2 &\longrightarrow 2 \\ 4 &\longrightarrow 3 \\ 6 &\longrightarrow 4 \\ &\vdots \\ 2n-2 &\longrightarrow n \end{aligned}$$

13a. Some prions can be seen.

13b. No Vulcans are illogical.

13c. Some math courses are not loads of fun.

14a. $(p \wedge \neg q) \to r$

14b. $q \leftrightarrow r \vee p$ or $q \leftrightarrow (r \vee p)$

14c. $\neg(\neg r \wedge \neg p)$

14d. $(p \wedge r) \vee (q \wedge s)$

15a.

p	q	$\neg(p \wedge \neg q)$
1	1	1
1	0	0
0	1	1
0	0	1

15b.

p	q	$q \vee (p \leftrightarrow \neg q)$
1	1	1
1	0	1
0	1	1
0	0	0

15c.

p	q	r	$p \to (q \vee r)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1