Math 101 Exam #3 Key (Spring 2023)

1 Argument is invalid:

p	q	$[(\neg p \to q)$	$\wedge$	$(\neg q)\,]$	$\to$	$(\neg p)$
1	1	1	0	0	1	0
1	0	1	1	1	0	0
0	1	1	0	0	1	1
0	0	0	0	1	1	1

2 Argument is valid:

p	q	r	$[(p \leftrightarrow q) \wedge (p \vee r) \wedge (q \to r)]$	$\to$	$(q \vee r)$
1	1	1	1	1	1
1	1	0	0	1	1
1	0	1	0	1	1
1	0	0	0	1	0
0	1	1	0	1	1
0	1	0	0	1	1
0	0	1	1	1	1
0	0	0	0	1	0

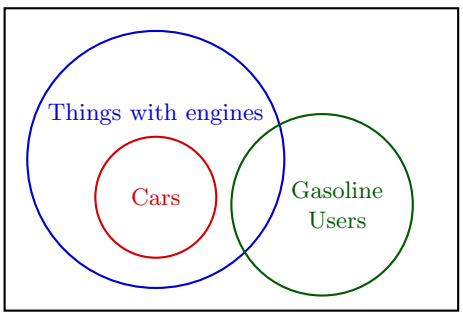
3 With $p =$"The soccer team wins the game," $q =$"Xavier played as goalkeeper," and $r =$"The team is in third place," the argument is:

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

The argument is valid:

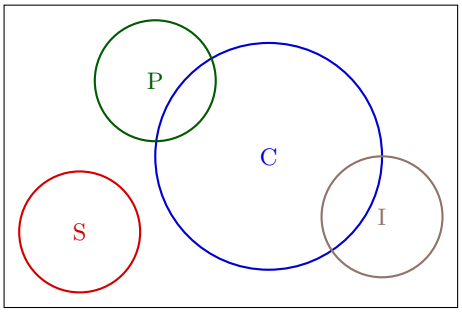
p	q	r	$[(p \to q) \wedge (q \to r)]$	$\to$	$(p \to r)$
1	1	1	1	1	1
1	1	0	0	1	0
1	0	1	0	1	1
1	0	0	0	1	0
0	1	1	1	1	1
0	1	0	0	1	1
0	0	1	1	1	1
0	0	0	1	1	1

4a



Invalid

4b Let P be the set of Pythagoreans, S the set of those who have squared the circle, C the set of those claiming to have squared the circle, and I the set of insane individuals.



Invalid

5 $\dfrac{277}{277+787} = \dfrac{277}{1064} \approx 0.260.$

6a 0 **6b** 1 **6c** $\dfrac{16}{52} = \dfrac{4}{13}$ **6d** $\dfrac{48}{52} = \dfrac{12}{13}$

7a Think of the entire square as having area 1, and add the areas of the four white regions. This will be the relevant probability.

$$P(\text{white area}) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{18} = \frac{11}{36}.$$

7b $P(\text{shaded or dotted area}) = 1 - P(\text{white area}) = 1 - \dfrac{11}{36} = \dfrac{25}{36}.$

8 11:5 against.

9 $\dfrac{3}{19+3} = \dfrac{3}{22}.$

10 $\text{Expected Value} = \dfrac{3}{8}(\$6) + \dfrac{2}{8}(-\$3) + \dfrac{2}{8}(\$0) + \dfrac{1}{8}(-\$9) = \$0.375.$

11a $\text{Expected Value} = \dfrac{1}{3200}(\$1995) + \dfrac{2}{3200}(\$495) + \dfrac{3197}{3200}(-\$5) = -\$4.0625$

11b $\text{Fair price} = \text{Expected Value} + \text{Cost to Play} = -\$4.0625 + \$5 = \$0.9375 \approx \$0.94.$