Math 101 Exam #2 Key (Spring 2023)

1a Some cosmologists do not think big.

1b All logicians can tie their shoelaces.

1c Some of the lights are on.

2a Captain Kirk is not beaming down to the planet if and only if the red shirts are not doomed.

2b If Captain Kirk is beaming down to the planet, then the red shirts are doomed or the dilithium crystals are not cracked.

3a $\neg r \rightarrow (p \vee q)$

3b $q \leftrightarrow (r \vee p)$

3c $\neg(\neg s \wedge \neg p)$

3d $(p \wedge r) \vee (q \wedge s)$

4a

p	q	$\neg(q \vee \neg p)$
1	1	0
1	0	1
0	1	0
0	0	0

4b

p	q	r	$\neg p \wedge (q \vee r)$
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

4c

p	q	$(\neg q \rightarrow p) \leftrightarrow \neg q$
1	1	0
1	0	1
0	1	0
0	0	0

5

p	q	$\neg(p \to \neg q)$	$p \wedge q$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	0	0

Equivalent

6 Let p be "Entropy always increases in an open thermodynamic system," and let q be "Radiocarbon dating is reliable." The given statement is then $\neg(p \vee \neg q)$, and by DeMorgan's Laws this becomes $\neg p \wedge q$, which translates as "Entropy does not always increase in an open thermodynamic system and radiocarbon dating is reliable." (This happens to be factually true, by the way, but it is not our concern.)

7 "Either the store is not out of Mint Milano cookies, or I am going to pitch a fit."

8 *Contrapositive:* "If it is raining, then the sky is cloudy."
Converse: "If it is not raining, then the sky is not cloudy."

9 Let

p: *The package was sent by Federal Express.*
q: *The package was sent by UPS.*
r: *The package arrived on time.*

Then

$$i:\ p \vee (\neg q \wedge r)$$
$$ii:\ r \leftrightarrow (p \vee \neg q)$$
$$iii:\ \neg p \to (\neg q \wedge r)$$

We can use a truth table to determine any logical equivalences.

p	q	r	$p \vee (\neg q \wedge r)$	$r \leftrightarrow (p \vee \neg q)$	$\neg p \to (\neg q \wedge r)$
1	1	1	1	1	1
1	1	0	1	0	1
1	0	1	1	1	1
1	0	0	1	0	1
0	1	1	0	0	0
0	1	0	0	1	0
0	0	1	1	1	1
0	0	0	0	0	0

From this table is can be seen that $i \Leftrightarrow iii$ is the only equivalency.