Math 101 Exam #3 Key (Spring 2021)

1 Argument is valid:

p	q	r	$[(p \wedge (q \vee r))$	$\wedge$	$(q \to r)]$	$\to$	$(p \wedge r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	0	0	0	0	1	1	0
0	1	1	0	0	1	1	0
0	1	0	0	0	0	1	0
0	0	1	0	0	1	1	0
0	0	0	0	0	1	1	0

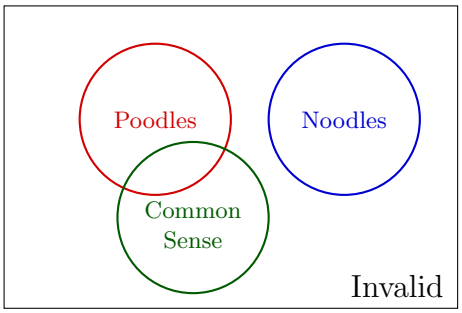
2 Let p be "The prescription was called in to Big Pharma Pill-o-Rama," and let q be "You can pick it up by tea time". The argument is:
$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore\ \neg p \end{array}$$
The argument is valid:

p	q	$[(p \to q)$	$\wedge$	$(\neg q)]$	$\to$	$(\neg p)$
1	1	1	0	0	1	0
1	0	0	0	1	1	0
0	1	1	0	0	1	1
0	0	1	1	1	1	1

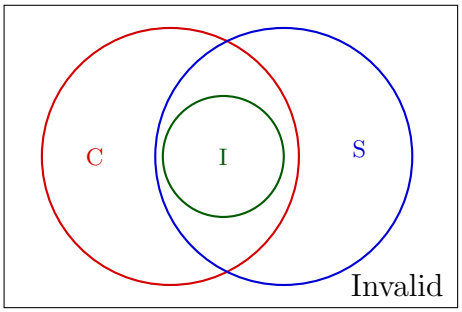
3 Let $p =$"Neroon wins the contest," let $q =$"Neroon will be rich," and let $r =$"Neroon will stop working." Argument:
$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ \neg r \to \neg p \end{array}$$
The argument is valid:

p	q	r	$[(p \to q)$	$\wedge$	$(q \to r)]$	$\to$	$(\neg r \to \neg p)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	1	0
1	0	1	0	0	1	1	1
1	0	0	0	0	1	1	0
0	1	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1

4a



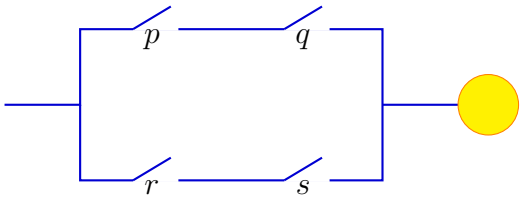
4b Let C be the set of circus clowns, S the set of scary things, and I the set of insurrectionists.



5 Statement: $p \vee \neg q \vee (r \wedge q)$.

p	q	r	$p \vee \neg q \vee (r \wedge q)$	Bulb
1	1	1	1	On
1	1	0	1	On
1	0	1	1	On
1	0	0	1	On
0	1	1	1	On
0	1	0	0	Off
0	0	1	1	On
0	0	0	1	On

6



7a $P(\text{cat}) = \dfrac{45}{56+45+12+7} = \dfrac{45}{120} = \dfrac{3}{8}$

7b $P(\text{ferret or dog}) = \dfrac{7+56}{56+45+12+7} = \dfrac{63}{120} = \dfrac{21}{40}$

8 $P(\text{yellow}) = \dfrac{5}{25+5+55} = \dfrac{5}{85} = \dfrac{1}{17}$

9 Probability $= P(4 \text{ or } 5 \text{ or } 6 \text{ or } 7 \text{ or } 8) = \dfrac{20}{52} = \dfrac{5}{13}$

10 $P(\text{not a } 5) = 1 - P(5) = 1 - \dfrac{4}{52} = \dfrac{48}{52} = \dfrac{12}{13}$

11 Odds against a number less than $3 = \dfrac{P(\text{no number less than } 3)}{P(\text{number less than } 3)} = \dfrac{4/6}{2/6} = \dfrac{4}{2}$, which translates as 4:2 against, or equivalently 2:1 against.

12 $P(\text{win funny hat}) = \dfrac{8}{13+8} = \dfrac{8}{21}$

13 Expected Value $= \dfrac{8}{16}(\$8) + \dfrac{2}{16}(-\$6) + \dfrac{4}{16}(-\$2) + \dfrac{1}{16}(-\$40) + \dfrac{1}{16}(\$0) = \$0.25.$

14a Expected Value $= \dfrac{1}{2000}(\$1197) + \dfrac{2}{2000}(\$597) + \dfrac{1997}{2000}(-\$3) = -\$1.80$

14b Fair price = Expected Value + Cost to Play $= -\$1.80 + \$3 = \$1.20$