Math 101 Exam #1 Key (Spring 2021)

1a $\{-2, -1, 0, 1, 2, 3\}$

1b $\{4, 5, 6, \ldots, 14\}$

1c $\{-1\}$, since $2x + 3 = 1 \quad \Rightarrow \quad x = -1$.

2a $\{x \mid x \in \mathbb{N} \text{ and } 4 \leq x \leq 12\}$

2b $\{x \mid x \in \mathbb{N} \text{ and } x \text{ is odd}\}$, or equivalently $\{2n - 1 \mid n \in \mathbb{N}\}$.

3a False, since it is # that is an element, and not {#}.

3b True.

3c False, ⧈ is an element of the set, not a proper subset of it.

4 $\varnothing$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$

5a $(A \cup B)' = \{0, 6\}$

5b $A' \cup (A \cap B) = \{0, 3, 6, 7\} \cup \{4\} = \{0, 3, 4, 6, 7\}$

5c $A - B' = \{1, 2, 4, 5, 8\} - \{0, 1, 2, 5, 6, 8\} = \{4\}$

6a Here $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$, and $C' = \{1, 2, 3, 4, 5, 6\}$, so

$$(C' \cup A) \cap B = \{2, 4, 6\}.$$

6b Note $A - B = A$, so $(A - B)' = A' = B$ and we have

$$(A - B)' - C = B - C = \{2, 4, 6, 8\} - \{7, 8, 9\} = \{2, 4, 6\}.$$

7 We have

$$B \times A = \{(4, q), (6, q), (8, q), (4, r), (6, r), (8, r)\},$$

and so $n(A) = 2$, $n(B) = 3$, and $n(B \times A) = n(B) \cdot n(A) = 6$.

8 We have

$$A \cap (B \cup C) = \{r, v, w, x, p, n, z\} \cap \{z, n, p, x, s, m, e, g, t, k\} = \{z, n, p, x\},$$

and

$$\begin{aligned}(A' \cup B) \cap C &= (\{t, k, g, e, m, s, j\} \cup \{t, k, g, p, n, z\}) \cap \{g, e, m, s, x, p\} \\ &= \{t, k, g, e, m, s, p, n, z, j\} \cap \{g, e, m, s, x, p\} \\ &= \{g, e, m, s, p\}\end{aligned}$$

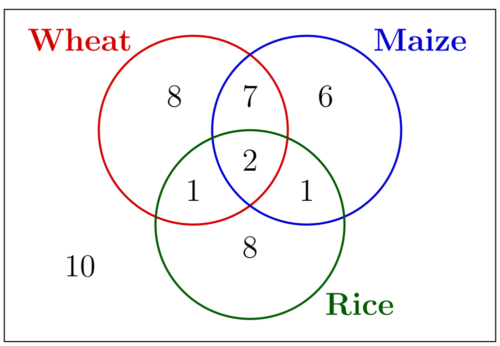
9a Referring to the regions of a two-set Venn diagram, we find the sets to be equal:

$$(A' \cap B)' = \{\text{I, II, IV}\} = A \cup B'.$$

9b Referring to the regions of a three-set Venn diagram, we find the sets to be not equal:

$$A \cup (B \cap C)' = \{\text{I, II, III, IV, V, VII, VIII}\} \neq \{\text{III, VI, VII}\} = A' \cap (B \cup C).$$

10a



10b $43 - 33 = 10$

10c $8 + 8 + 6 = 22$

10d 7

10e $8 + 1 + 6 = 15$

11

$$\begin{aligned} 5 &\longrightarrow 9 \\ 9 &\longrightarrow 13 \\ 13 &\longrightarrow 17 \\ 17 &\longrightarrow 21 \\ &\vdots \\ 4n+1 &\longrightarrow 4n+5 \\ &\vdots \end{aligned}$$