# Math 101 Exam #3 Key (Fall 2022)

**1** Argument is valid:

| $p$ | $q$ | $r$ | $[(p \wedge (q \vee r))$ | $\wedge$ | $(q \to r)]$ | $\to$ | $(p \wedge r)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |

**2** Let $p$ be "The prescription was called in to Big Pharma Pill-o-Rama," and let $q$ be "You can pick it up by tea time". The argument is: $p \to q$, $\neg q$, $\therefore\ \neg p$

The argument is valid:

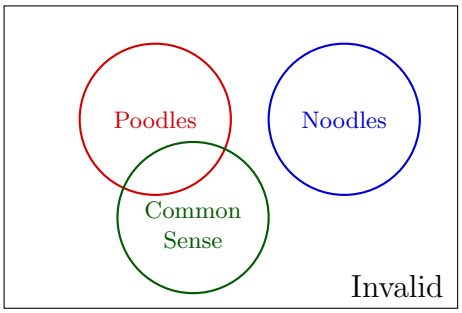
| $p$ | $q$ | $[(p \to q)$ | $\wedge$ | $(\neg q)]$ | $\to$ | $(\neg p)$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |

**3** Let $p =$"Neroon wins the contest," let $q =$"Neroon will be rich," and let $r =$"Neroon will stop working." Argument: $p \to q$, $q \to r$, $\therefore\ \neg r \to \neg p$

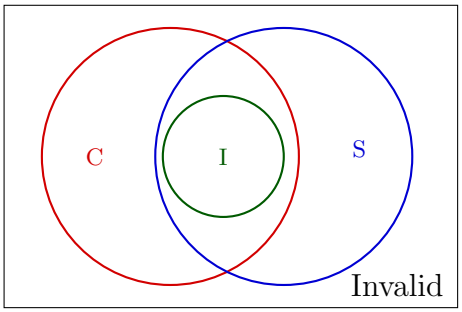
The argument is valid:

| $p$ | $q$ | $r$ | $[(p \to q)$ | $\wedge$ | $(q \to r)]$ | $\to$ | $(\neg r \to \neg p)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |

**4a**



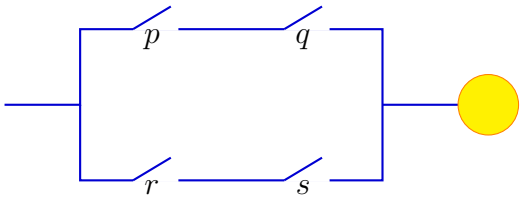

**4b** Let $C$ be the set of circus clowns, $S$ the set of scary things, and $I$ the set of insurrectionists.




**5** Statement: $p \vee \neg q \vee (r \wedge q)$.

| $p$ | $q$ | $r$ | $p \vee \neg q \vee (r \wedge q)$ | Bulb |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | On |
| 1 | 1 | 0 | 1 | On |
| 1 | 0 | 1 | 1 | On |
| 1 | 0 | 0 | 1 | On |
| 0 | 1 | 1 | 1 | On |
| 0 | 1 | 0 | 0 | Off |
| 0 | 0 | 1 | 1 | On |
| 0 | 0 | 0 | 1 | On |

**6**




**7a** $P(\text{cat}) = \dfrac{45}{56+45+12+7} = \dfrac{45}{120} = \dfrac{3}{8}$

**7b** $P(\text{ferret or dog}) = \dfrac{7+56}{56+45+12+7} = \dfrac{63}{120} = \dfrac{21}{40}$

**8** $P(\text{yellow}) = \dfrac{5}{25+5+55} = \dfrac{5}{85} = \dfrac{1}{17}$

**9** Probability $= P(1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5 \text{ or J or Q or K}) = \dfrac{32}{52} = \dfrac{8}{13}$

**10** $P(\text{not a } 5) = 1 - P(5) = 1 - \dfrac{4}{52} = \dfrac{48}{52} = \dfrac{12}{13}$

**11** Odds against a number less than $3 = \dfrac{P(\text{no number less than } 3)}{P(\text{number less than } 3)} = \dfrac{4/6}{2/6} = \dfrac{4}{2}$, which translates as 4:2 against, or equivalently 2:1 against.

**12** $P(\text{win funny hat}) = \dfrac{16}{9+16} = \dfrac{16}{25}$

**13** Expected Value $= \dfrac{8}{16}(\$8) + \dfrac{2}{16}(-\$6) + \dfrac{4}{16}(-\$2) + \dfrac{1}{16}(-\$40) + \dfrac{1}{16}(\$0) = \$0.25.$

**14a** Expected Value $= \dfrac{1}{2000}(\$1197) + \dfrac{2}{2000}(\$597) + \dfrac{1997}{2000}(-\$3) = -\$1.80$

**14b** Fair price = Expected Value + Cost to Play $= -\$1.80 + \$3 = \$1.20$