Math 101 Exam #2 Key (Fall 2022)

1a Some cosmologists do not think big.

1b All logicians can tie their shoelaces.

1c All who wander are lost.

2a Captain Kirk is not beaming down to the planet if and only if the red shirts are not doomed.

2b If Captain Kirk is beaming down to the planet, then the red shirts are doomed or the dilithium crystals are not cracked.

3a $\neg r \to (p \vee q)$

3b $q \leftrightarrow (r \vee p)$

3c $\neg(\neg s \wedge \neg p)$

3d $(p \wedge r) \vee (q \wedge s)$

4a

p	q	$\neg(q \vee \neg p)$
1	1	0
1	0	1
0	1	0
0	0	0

4b

p	q	r	$\neg p \wedge (q \vee r)$
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

4c

p	q	$(\neg q \to p) \leftrightarrow \neg q$
1	1	0
1	0	1
0	1	0
0	0	0

5

p	q	$\neg(p \to \neg q)$	$p \wedge q$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	0	0

Equivalent

6 Let p be "Entropy always increases in an open thermodynamic system," and let q be "Radiocarbon dating is reliable." The given statement is then $\neg(p \vee \neg q)$, and by DeMorgan's Laws this becomes $\neg p \wedge q$, which translates as "Entropy does not always increase in an open thermodynamic system and radiocarbon dating is reliable." (This happens to be factually true, by the way, but it is not our concern.)

7 Let p be "The clowns in Congress will listen to the people," and q be "The system does work." Given statement is thus $p \vee \neg q$, which by the given equivalency becomes $\neg p \to \neg q$ and translates as "If the clowns in Congress don't listen to the people, then the system doesn't work."

8 *Contrapositive:* "If Lieutenant Tragg will need an antacid, then Perry Mason has arrived at the scene of the crime."

Converse: "If Lieutenant Tragg will not need an antacid, then Perry Mason has not arrived at the scene of the crime."

9 Let

p: *The package was sent by Federal Express.*
q: *The package was sent by UPS.*
r: *The package arrived on time.*

Then

$$\begin{aligned} i&: \ p \vee (\neg q \wedge r) \\ ii&: \ r \leftrightarrow (p \vee \neg q) \\ iii&: \ \neg p \to (\neg q \wedge r) \end{aligned}$$

We can use a truth table to determine any logical equivalences.

p	q	r	$p \vee (\neg q \wedge r)$	$r \leftrightarrow (p \vee \neg q)$	$\neg p \to (\neg q \wedge r)$
1	1	1	1	1	1
1	1	0	1	0	1
1	0	1	1	1	1
1	0	0	1	0	1
0	1	1	0	0	0
0	1	0	0	1	0
0	0	1	1	1	1
0	0	0	0	0	0

From this table is can be seen that $i \Leftrightarrow iii$ is the only equivalency.