## Math 101 Exam #1 Key (Fall 2022)

1c 
$$\{1\}$$
, since  $3 - 2x = 1$  implies  $x = 1$ .

**2a** 
$$\{x \mid x \in \mathbb{N} \text{ and } 4 \le x \le 12\}$$

**2b** 
$$\{3n \mid n \in \mathbb{N}\}$$
, or equivalently  $\{n \mid n/3 \in \mathbb{N}\}$ .

**3a** False, since it is 
$$\#$$
 that is an element, and not  $\{\#\}$ .

**4** 
$$\varnothing$$
,  $\{a\}$ ,  $\{b\}$ ,  $\{c\}$ ,  $\{a,b\}$ ,  $\{a,c\}$ ,  $\{b,c\}$ 

**5a** 
$$(A \cup B)' = \{0, 6\}$$

**5b** 
$$A' \cup (A \cap B) = \{0, 3, 6, 7\} \cup \{4\} = \{0, 3, 4, 6, 7\}$$

**5c** 
$$A - B' = \{1, 2, 4, 5, 8\} - \{0, 1, 2, 5, 6, 8\} = \{4\}$$

**6a** Here 
$$A = \{1, 3, 5, 7, 9\}$$
,  $B = \{2, 4, 6, 8\}$ , and  $C' = \{1, 2, 3, 4, 5, 6\}$ , so 
$$(C' \cup A) \cap B = \{2, 4, 6\}.$$

**6b** Note 
$$A - B = A$$
, so  $(A - B)' = A' = B$  and we have 
$$(A - B)' - C = B - C = \{2, 4, 6, 8\} - \{7, 8, 9\} = \{2, 4, 6\}.$$

7 We have

$$A \times B = \{(s,4), (s,6), (s,8), (t,4), (t,6), (t,8)\},\$$

and so 
$$n(A) = 2$$
,  $n(B) = 3$ , and  $n(A \times B) = n(A) \cdot n(B) = 6$ .

8 We have

$$A \cap (B \cup C) = \{r, v, w, x, p, n, z\} \cap \{z, n, p, x, s, m, e, g, t, k\} = \{z, n, p, x\},\$$

and

$$(A' \cup B) \cap C = (\{t, k, g, e, m, s, j\} \cup \{t, k, g, p, n, z\}) \cap \{g, e, m, s, x, p\}$$
$$= \{t, k, g, e, m, s, p, n, z, j\} \cap \{g, e, m, s, x, p\}$$
$$= \{g, e, m, s, p\}$$

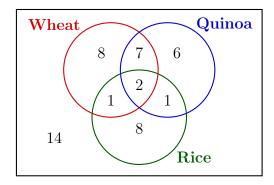
**9a** Referring to the regions of a two-set Venn diagram, we find the sets to be equal:

$$(A' \cap B)' = \{ I, II, IV \} = A \cup B'.$$

**9b** Referring to the regions of a three-set Venn diagram, we find the sets to be not equal:

$$A \cup (B \cap C)' = \{I, II, III, IV, V, VII, VIII\} \neq \{III, VI, VII\} = A' \cap (B \cup C).$$

10a



**10b** 
$$47 - 33 = 14$$

**10c** 
$$7+1+1=9$$

**10d** 7

**10e** 
$$8+1+6=15$$

$$\begin{array}{c}
-2 \longrightarrow 4 \\
4 \longrightarrow 10 \\
10 \longrightarrow 16 \\
16 \longrightarrow 22 \\
\vdots \\
6n - 8 \longrightarrow 6n - 2 \\
\vdots$$