Math 101 Exam #4 Key (Fall 2012)

1 $\frac{16}{52} = \frac{4}{13} \approx 0.308$

2a $\frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169} \approx 0.0059$

2b $\frac{4}{52} \cdot \frac{4}{51} = \frac{1}{13} \cdot \frac{4}{51} = \frac{4}{663} \approx 0.0060$

3 $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{81}{4096} \approx 0.020$

4a $P(\text{0-60} \mid \text{male}) = \frac{41}{171} \approx 0.240$

4b $P(\text{female} \mid \text{over 120}) = \frac{14}{35} = \frac{2}{5} \approx 0.400$

4c $P(\text{61-120 or over 120} \mid \text{male}) = P(\text{61-120} \mid \text{male}) + P(\text{over 120} \mid \text{male}) = \frac{109}{171} + \frac{21}{171} = \frac{130}{171} \approx 0.760$

5a $26 \cdot 25 \cdot 24 \cdot 23 \cdot 10 \cdot 9 = 32,292,000$

5b $26^4 \cdot 10^2 = 45,697,600$

6 $3^{10} = 59,049$

7 17 letters in all, with 3 A's, 3 R's, 2 M's, and 2 U's: $\dfrac{17!}{(3!)(3!)(2!)(2!)} \approx 2.47005 \times 10^{12}$

8 ${}_{26}\mathrm{C}_{18} = 1,562,275$

9 ${}_{12}\mathrm{C}_5 \cdot {}_8\mathrm{C}_3 = 792 \cdot 56 = 44,352$

10 $\dfrac{{}_9\mathrm{C}_4}{{}_{16}\mathrm{C}_4} = \dfrac{126}{1820} = \dfrac{9}{130} \approx 0.0692$

11a $\dfrac{{}_2\mathrm{C}_2}{{}_7\mathrm{C}_2} = \dfrac{1}{21} \approx 0.048$

11b $P(\text{at least 1 car}) = P(\text{1 car or 2 cars}) = P(\text{1 car}) + P(\text{2 cars}) = \dfrac{{}_2\mathrm{C}_1 \cdot {}_5\mathrm{C}_1}{{}_7\mathrm{C}_2} + \dfrac{{}_2\mathrm{C}_2}{{}_7\mathrm{C}_2} = \frac{10}{21} + \frac{1}{21} = \frac{11}{21} \approx 0.524$

12 $\dfrac{{}_4\mathrm{C}_2 \cdot {}_4\mathrm{C}_2 \cdot {}_{44}\mathrm{C}_1}{{}_{52}\mathrm{C}_5} = \dfrac{(6)(6)(44)}{2,598,960} = \dfrac{1,584}{2,598,960} = \dfrac{33}{54,145} \approx 0.000609$

13 $P(2) = ({}_{14}\mathrm{C}_2)(0.005)^2(0.995)^{12} = 91(0.005)^2(0.995)^{12} \approx 0.002142$

14a $P(5) = ({}_{10}\mathrm{C}_5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^5 = 252(0.2)^5(0.8)^5 \approx 0.0264$

14b $P(9)+P(10) = ({}_{10}\mathrm{C}_9)\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)^1+({}_{10}\mathrm{C}_{10})\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^0 = 10\left(\frac{1}{5}\right)^9\left(\frac{4}{5}\right)+\left(\frac{1}{5}\right)^{10} = 4.1984\times10^{-6}$

14c $P(\geq 1) = 1 - P(0) = 1 - ({}_{10}\mathrm{C}_0)\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10} = 1 - \left(\frac{4}{5}\right)^{10} \approx 1 - 0.107 \approx 0.893$