Math 101 Exam #2 Key (Fall 2012)

1 For n any natural number we have

$$6 \longrightarrow 11$$
$$11 \longrightarrow 16$$
$$16 \longrightarrow 21$$
$$21 \longrightarrow 26$$
$$\vdots$$
$$5n + 1 \longrightarrow 5n + 6$$

2 For n any natural number we have

$$\tfrac{1}{3} \longrightarrow 1$$
$$\tfrac{1}{6} \longrightarrow 2$$
$$\tfrac{1}{9} \longrightarrow 3$$
$$\tfrac{1}{12} \longrightarrow 4$$
$$\vdots$$
$$\tfrac{1}{3n} \longrightarrow n$$

3a Some particle physicists do not think small.

3b All mathematicians write rigorous proofs.

3c Some tax decreases pay for themselves.

4a Conservative viewpoints do not always lose in the long run if and only if slavery is accepted.

4b If conservative viewpoints always lose in the long run, then slavery is not accepted and child labor is not legal.

5a $\neg r \to (p \vee q)$

5b $q \leftrightarrow (r \vee p)$

5c $\neg(\neg s \wedge \neg p)$

5d $(p \wedge r) \vee (q \wedge s)$

6a

p	q	$\neg(q \vee \neg p)$
1	1	0
1	0	1
0	1	0
0	0	0

6b

p	q	r	$\neg p \wedge (q \vee r)$
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

6c

p	q	$(\neg q \to p) \leftrightarrow \neg q$
1	1	0
1	0	1
0	1	0
0	0	0

7

p	q	$\neg(p \to \neg q)$	$p \wedge q$	
1	1	1	1	Equivalent
1	0	0	0	
0	1	0	0	
0	0	0	0	

8. Let p be "Entropy always increases in an open thermodynamic system," and let q be "Radiocarbon dating is reliable." The given statement is then $\neg(p \vee \neg q)$, and by DeMorgan's Laws this becomes $\neg p \wedge q$, which translates as "Entropy does not always increase in an open thermodynamic system and radiocarbon dating is reliable." (This happens to be factually true, by the way, but it is not our concern.)

9. Let p be "The clowns in Congress will listen to the people," and q be "The system does work." Given statement is thus $p \vee \neg q$, which by the given equivalency becomes $\neg p \to \neg q$ and translates as "If the clowns in Congress don't listen to the people, then the system doesn't work."

10. *Contrapositive:* "If you do not have something incriminating to hide, then you do not refuse to release all your tax returns until after election day."

  Converse: "If you have something incriminating to hide, then you refuse to release all your tax returns until after election day."

11 Let

   p: *The package was sent by Federal Express.*
   q: *The package was sent by UPS.*
   r: *The package arrived on time.*

Then

$$i: \ p \vee (\neg q \wedge r)$$
$$ii: \ r \leftrightarrow (p \vee \neg q)$$
$$iii: \ \neg p \rightarrow (\neg q \wedge r)$$

We can use a truth table to determine any logical equivalences.

p	q	r	$p \vee (\neg q \wedge r)$	$r \leftrightarrow (p \vee \neg q)$	$\neg p \rightarrow (\neg q \wedge r)$
1	1	1	1	1	1
1	1	0	1	0	1
1	0	1	1	1	1
1	0	0	1	0	1
0	1	1	0	0	0
0	1	0	0	1	0
0	0	1	1	1	1
0	0	0	0	0	0

From this table is can be seen that $i \Leftrightarrow iii$ is the only equivalency.